On the Conceptual Bases of Time Reversal and Orthodox QED

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Provided that time is a parameter in Newtonian classical mechanics but a coordinate in Einsteinian relativistic mechanics, we elaborate on the differences of time inversion in the classical and relativistic settings. The comprehension of these differences is crucial to improving orthodox QED.

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1. Classical Time Reversal

Newtonian classical mechanics is established on a three-dimensional Euclidean space with time as a parameter. Classical time reversal is based on a motion picture: when time t changes to $-t$, velocity $\mathbf{v}$ goes to $-\mathbf{v}$, since $\mathbf{v} = d\mathbf{x}/dt$, and thus space momentum $\mathbf{p}$ to $-\mathbf{p}$, energy E to E. In general, any classical physical quantities being odd order in time or time derivative, change sign under time reversal, while other quantities being even order in time do not. Now that time plays a role of "parameter" here, it is adequate to call this type of reversal "classical parameter time reversal".

Based on this motion picture, an antiunitary time inversion (also called time reversal) has been established since the early days of the development of quantum mechanics [Refs.1,2,3]. In this standard approach, energy or Hamiltonian in a quantum mechanical system, is considered as a scalar, or an "invariant" under the time reversal. This consideration therefore determines the unique antiunitarity of time reversal. With a closer inspection, however, it is not hard to realize that this approach is inherently classical or "non-relativistic", since it stems from a classical concept not applicable in relativity (to become evident in the later discussions).

In classical quantum mechanics, the governing equation is the Schrödinger equation. To solve the Schrödinger equation, one needs to find a Hamiltonian for each specific problem. If one finds a time-reversal-violating term in the Hamiltonian, then one would draw a conclusion that time-reversal invariance is violated. Similarly one may look for other physical quantities, odd order in time, that may signal time-reversal symmetry breaking. In these cases, time is solely regarded as a parameter, and physical quantities evolve deterministically with the time parameter.

In Newtonian mechanics, nothing can go backward in time, and time is not really reversible, since there is only one clock running forward. Two typical examples are in order. Doing a parameter time reversal to reverse a motion, is like doing a space reversal with only one clock ticking forward. Running a movie backward is like trying to recall what happened yesterday, while the clock is still ticking forward. "Time goes on" is how we perceive the ever-changing world in a classical sense. This deterministic point of view on time parameter, so dominant for two hundrend years, encountered real challenge at the beginning of the twentieth century.

2. Relativistic Time Inversion

Einsteinian relativistic mechanics (restricted to special relativity for now) is established on a four-dimensional Minkowski space-time with time being an independent coordinate [Ref.4]. The Maxwell equations on electromagnetic fields can be well expressed in a tensor form incorporating space and time on an equal footing. The geometrical transition from 3d space to 4d space-time has become one of the cornerstones in relativity theory. With this transition, time starts to play a different role as a "coordinate", and becomes invertible as one sets up another clock running backward along time axis.

On 4d Minkowski space-time, energy and momentum form a four-vector momentum $(E, \mathbf{p})$ that transforms to $(-E, \mathbf{p})$ under time inversion, since energy, the time component, is a "time vector" along time axis [Ref.5]. To illustrate this, we may express the four-vector momentum in operator tensor form, $P_\mu = i\partial_\mu$, in which it is energy that is related to the first-order time derivative, rather than space momentum. It is also possible to find more physical quantities being odd (or even) in time classically, turn out to be even (or odd) in time when expressed in tensor forms, and thus behave differently under the time inversion. It is fitting to call this "relativistic coordinate time inversion".

In special relativity, the Lorentz group is the fundamental symmetry group of the Minkowski space-time, and the coordinate time inversion is just part of the Lorentz transformations. Introducing this generic coordinate time inversion in quantum mechanics, it is straightforward to show it is a unitary transformation. The unitary time inversion works with the other Lorentz transformations in a more natural and coherent way.

In relativistic quantum mechanics, the true governing equation is the Dirac equation, which should remain invariant under the Lorentz transformations. However, it is not difficult to show that the Dirac equation with the external electromagnetic potential, is not invariant under unitary time inversion. Due to this non-invariance, there appear a "Klein paradox" in the Dirac equation, and the subsequent infinity difficulties in quantum electrodynamics.

As mentioned earlier, nothing can go backward in time in a classical sense. But that does not preclude the possibility of moving backward in time in a relativistic sense, when speed approaches that of light. Traveling back in time has become a popular theme of science fiction, pending further investigation. One thing is evident nevertheless: within Einstein's relativity theory, time is not what it used to be.

3. On Orthodox QED

Up to this point, one may still wonder why bother to talk about the differences of time inversion in the classical and relativistic theories. After all, time inversion means changing t to $-t$, nothing too complex to comprehend. This sentiment would have been fully legitimate, had our highly successful theories not run into serious trouble.

Despite excellent agreements with experiments, QED, based on the Dirac and Maxwell equations, has been plagued by infinities since the very beginning. To do away with infinities, many efforts have been made in the development of renormalization. There are pros and cons about this *ad hoc* program. To avoid getting carried away by debating over renormalization, we should at least look harder into the foundations of QED and ask this question: do we have other alternatives that would do better to cure the pathologies of QED?

QED is based on three fundamental principles: special relativity, quantum mechanics and gauge invariance. However, antiunitary time reversal, stemming from the classical motion picture in Newtonian mechanics, is not consistent with special relativity. To establish self-consistent quantum theories in the relativistic framework, it is imperative to replace the classical antiunitary time reversal with the relativistic unitary time inversion. This replacement would generate a nonlinear electrodynamical interaction potential in the Dirac equation, and thus resolve the long-standing "Klein paradox", and eventually lead to a nonsingular nonlinear QED [Ref.5].

Any physical theory has its own limited range of applicability. Newtonian classical mechanics cannot be extended to Einsteinian relativistic mechanics without fundamental changes in physical concepts. One of the most significant changes in this regard is the role played by time: from parameter to coordinate. Although time inversion means changing t to $-t$, it has different consequences in the classical and relativistic settings, as outlined hereinabove. Ignoring these differences would render relativistic quantum theories like orthodox QED full of loopholes that are hard to fix at a fundamental level.

For the very same reason, it should also be mentioned that the unitary time inversion is applicable only on a local flat Minkowski space-time, where the gravitational effect is negligible. On a global curved space-time, time inversion may not be in the symmetry group of that space-time in general relativity, and its physical consequences require new insights.

References

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